Accidental Symmetries and N=1 Duality
in Supersymmetric Gauge Theory

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We note that the accidental symmetries which are present in some examples of duality imply the existence of continuously infinite sets of theories with the same infrared behavior. These sets interpolate between theories of different flavors and colors; the change in color and flavor is compensated by interactions (often non-perturbative) induced by operators in the superpotential. As an example we study the behavior of $SU(2)$ gauge theories with $2N_f$ doublets; these are dual to $SU(N_f - 2)$ gauge theories whose ultraviolet flavor symmetry is $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ but whose flavor symmetry is $SU(2N_f)$ in the infrared. The infrared $SU(2N_f)$ flavor symmetry is implemented in the ultraviolet as a non-trivial transformation on the Lagrangian and matter content of the magnetic theory, involving (generally non-renormalizable) baryon operators and non-perturbative dynamics. We discuss various implications of this fact, including possible new chiral fixed points and interesting examples of dangerously irrelevant operators.

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1. Introduction

Recent work has uncovered many interesting aspects of four-dimensional N=1 super-symmetric gauge theories. Foremost is the electric-magnetic duality discovered by Seiberg [1]. Completely different, yet equivalent, descriptions may exist for a given field theory. It is apparent that formerly sacred principles must be reconsidered; for example the gauge group does not so much *define* a model, as give a (perhaps weakly-coupled) description of that model. Global symmetry however is believed to encode aspects of the physical content of the theory. Many theories which are dual to one another share the same global symmetries in the ultraviolet (UV) as well as in the infrared (IR). However, the matching of global symmetries need not occur at the level of the perturbative definition of two dual theories; rather, it need only occur in the far IR. Many models exist for which the global symmetries of the UV descriptions are different.

In this paper, we explore some consequences of this fact. We consider the simple example of $SU(2)$ gauge theories with $2N_f$ doublets, which have a dual description in terms of $SU(N_f - 2)$ gauge theories that have N_f flavors and a smaller global symmetry. We show how the latter theories have an "accidentally" enhanced global symmetry in the IR, through a mixture of perturbative and non-perturbative physics. Using this we uncover non-trivial continuously infinite classes of theories which all flow to the same IR theory. We also discuss a number of interesting issues which are raised by our discussion, including examples of dangerously irrelevant operators and new chiral fixed points.

2. Baryons in $SU(N_c)$ Gauge Theories

We begin with some introductory material on N=1 supersymmetric $SU(N_c)$ gauge theory, paying particular attention to the properties of baryon operators under the duality mapping. The dynamics studied here will be used in Sec. 3.

We consider an $SU(N_c)$ supersymmetric gauge theory with N_f flavors of chiral su-perfields $Q^{\alpha r}$ in the $\mathbf{N}_c$ and $\tilde{Q}_{\alpha u}$ in the $\overline{\mathbf{N}}_c$ of $SU(N_c)$. We will refer to this as the electric theory. This model has an $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ global symmetry.[3] According to the duality proposed by Seiberg [1], the magnetic theory is $SU(N_f - N_c)$ with N_f flavors q_r^α and $\tilde{q}_\alpha^u$, along with gauge singlet superfields $M_u^{\ r}$ and a superpotential

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[3] Throughout the paper we use the letters α, β for color indices, $r, s, \ldots$ for $SU(N_f)_L$ and $u, v, \ldots$ for $SU(N_f)_R$.

$W = \mu^{-1} q \cdot M \cdot \tilde{q}$; here μ is a scale factor needed to match the dimensionful quantities of the magnetic theory to those of the electric theory [2,3,4,5].[4] The magnetic theory also possesses a global $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ symmetry; note that the fields Q and q transform differently under this symmetry. If the electric gauge group is $SU(2)$, then the flavor symmetry is enlarged to $SU(2N_f)$. However, the flavor symmetry in its dual remains $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$; the full $SU(2N_f)$ symmetry is realized only in the IR as an accidental symmetry.

The gauge-invariant chiral operators of the electric theory are mapped under the duality to operators in the magnetic theory. In Ref. [1], it was checked that perturbations involving superpotential deformations by and expectation values for meson operators preserve the duality. It may be checked that the same is true for baryon operators, as was done in part in [6].

We will refer to the invariant operators of the theory in the following way. The mesons $M_u{}^r$ of the theory are $M(Q) \equiv Q\tilde{Q}$ in the electric theory and are fundamental fields M in the magnetic dual. The baryons $B^{i_1 \dots i_{N_c}}$ are

$$B^{r_1 \dots r_{N_c}}(Q) = \epsilon_{\alpha_1 \dots \alpha_{N_c}} Q^{\alpha_1 r_1} \dots Q^{\alpha_{N_c} r_{N_c}} \tag{2.1}$$

in the electric theory and

$$B^{r_1 \dots r_{N_c}}(q) = \sqrt{\frac{-\Lambda^b}{(-\mu)^{N_f - N_c}}} \frac{1}{(N_f - N_c)!} \epsilon^{r_1 \dots r_{N_f}} \epsilon^{\beta_1 \dots \beta_{(N_f - N_c)}} q_{\beta_1 r_{N_c+1}} \dots q_{\beta_{(N_f - N_c)} r_{N_f}} \tag{2.2}$$

in the magnetic theory [4]. Analogous formulas hold for the antibaryons.

We now consider a general $SU(N_c)$ theory with $N_f \geq N_c + 2$ flavors with a superpotential which contains baryon operators, and study its flow and that of its dual when mass terms are added or symmetries are broken. (Some of this discussion also appears in Ref. [6].) We begin by comparing the flat directions in the two theories.

Seiberg [1] showed that the flat directions of the theory and its dual match in the absence of a superpotential. Some of these directions are lifted by the superpotential

$$W_{\text{tree}} = \frac{1}{N_c!} \left[h_{r_1 \dots r_{N_c}} B^{r_1 \dots r_{N_c}}(Q) + \tilde{h}^{u_1 \dots u_{N_c}} \tilde{B}_{u_1 \dots u_{N_c}}(\tilde{Q}) \right]. \tag{2.3}$$

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[4] Specifically, the holomorphic dynamical scales Λ and $\tilde{\Lambda}$ of the electric and magnetic theories are related by $\Lambda^b \tilde{\Lambda}^{\tilde{b}} = (-)^{N_f - N_c} \mu^{N_f}$, where $b = 3N_c - N_f$ and $\tilde{b} = \tilde{N}_c - N_f = 2N_f - 3N_c$ are the coefficients of the one-loop beta function in the two theories.

From the F-flatness conditions $\langle \frac{\partial W}{\partial Q^r} \rangle = 0$ one finds

$$\langle Q^s \frac{\partial W}{\partial Q^r} \rangle \propto \langle h_{rr_2 \ldots r_{N_c}} B^{sr_2 \ldots r_{N_c}} \rangle = 0 \tag{2.4}$$

for all r, s, and similar conditions on $\tilde{h}\tilde{B}$.

In the magnetic theory

$$W_{\text{tree}} = \mu^{-1} M_u^r q_r \tilde{q}^u + \frac{1}{N_c!} \left[h_{r_1 \ldots r_{N_c}} B^{r_1 \ldots r_{N_c}}(q) + \tilde{h}^{u_1 \ldots u_{N_c}} \tilde{B}_{u_1 \ldots u_{N_c}}(\tilde{q}) \right] \tag{2.5}$$

the flatness conditions are

$$\langle \frac{\partial W}{\partial q_r} \rangle = \langle \frac{1}{N_c!} h_{r_1 \ldots r_{N_c}} \frac{\partial}{\partial q_r} B^{r_1 \ldots r_{N_c}}(q) + \frac{1}{\mu} M_u^r \tilde{q}^u \rangle = 0$$

$$\langle \frac{1}{N_c!} \tilde{h}^{u_1 \ldots u_{N_c}} \frac{\partial}{\partial \tilde{q}^u} \tilde{B}_{u_1 \ldots u_{N_c}}(\tilde{q}) + \frac{1}{\mu} M_u^r q_r \rangle = 0 \tag{2.6}$$

$$\langle q_r \tilde{q}^u \rangle = 0 \; .$$

Multiplying the first equation of (2.6) by q_s and applying the third equation, we find the condition (2.4). The condition on the antibaryon is similarly recovered.

There are also restrictions on the expectation values of the operators M_u^r. We consider the simplest case, when all components of $h, \tilde{h}$ are zero except $h_{12 \ldots N_c}$ (and its permutations). The F-flatness conditions, which force the $N_c \times N_c$ matrix $Q^{\alpha r}$ to have rank $N_c - 2$ or less, impose a similar constraint on the matrix $M_u^{\bar{r}}(Q)$, where $\bar{r} = 1, \cdots, N_c$ and $u = 1, \cdots, N_f$. To see this in the magnetic theory, let us first note that there is no constraint on M_u^r from (2.6) if all q and $\tilde{q}$ are zero. However, as in [1], a dynamical superpotential is generated if M is rank $N_c - 1$. For simplicity let M be diagonal with only $\langle M_1^1 \rangle, \langle M_2^2 \rangle, \cdots \langle M_{N_c-1}^{N_c-1} \rangle$ non-vanishing. Such a choice gives mass to the fields $q_r, \tilde{q}^u$, $r, u = 1, \cdots, N_c - 1$. When they are integrated out the theory confines [7,1], and the superpotential is a function of the singlet superfields $M_{\hat{u}}^{\hat{r}}$, $N_{\hat{r}}^{\hat{u}} = q_{\hat{r}} \tilde{q}^{\hat{u}}$, $B_L^{\hat{r}} \propto B^{12 \ldots (N_c-1)\hat{r}}(q)$ and $\tilde{B}_{L\hat{u}} \propto \tilde{B}_{12 \ldots (N_c-1)\hat{u}}(\tilde{q})$, where $\hat{r}, \hat{u} = N_c, N_c + 1, \ldots, N_f$.

$$W(M, N, B, \tilde{B}) = \mu^{-1} M_{\hat{u}}^{\hat{r}} N_{\hat{r}}^{\hat{u}} + (h_L)_{N_c} B_L^{N_c} - \sum_{a=1}^{N_c-1} \frac{N_{\hat{r}}^{\hat{u}} M_a^{\hat{r}} M_{\hat{u}}^a}{\mu M_a^a}$$

$$- \frac{\det(N_{\hat{r}}^{\hat{u}})}{\tilde{\Lambda}_L^{\tilde{b}_L}} - \frac{B_L^{\hat{r}} N_{\hat{r}}^{\hat{u}} \tilde{B}_{L\hat{u}}}{\mu} \; . \tag{2.7}$$

(The last two terms are due to strong coupling effects [7] while the term $N_{\hat{r}}^{\hat{u}} M_a^{\hat{r}} M_{\hat{u}}^a / \mu M_a^a$ is induced at tree level by integrating out the massive $q_r, \tilde{q}^u$ fields.) Here $(h_L)_{N_c} =$

$\langle Q^1 Q^2 \ldots Q^{N_c-1} \rangle h_{12\ldots N_c} = \sqrt{\langle M_1^1 M_2^2 \ldots M_{N_c-1}^{N_c-1} \rangle} h_{12\ldots N_c}$ is the low-energy baryon coupling, and the formula $\tilde{\Lambda}_L^{\tilde{b}_L} = \langle M_1^1 M_2^2 \ldots M_{N_c-1}^{N_c-1} \rangle \mu^{1-N_c} \tilde{\Lambda}^{\tilde{b}}$ relates the dynamical scales of the high- and low-energy magnetic theories. The conditions

$$\langle \frac{\partial W}{\partial M_{\hat{u}}^{N_c}} \rangle \propto \langle N_{N_c}^{\hat{u}} \rangle = 0$$

$$\langle \frac{\partial W}{\partial B_L^{N_c}} \rangle = (h_L)_{N_c} - \langle N_{N_c}^{\hat{u}} \tilde{B}_{L\hat{u}} \rangle \mu^{-1} = 0 \tag{2.8}$$

are inconsistent. This confirms that those flat directions with rank $(\langle M_u^{\bar{r}} \rangle) > N_c - 2$ are lifted by confinement effects. We will repeatedly encounter similar situations in which flat directions are "dynamically blocked" by the baryon operators in the superpotential.

We have shown here that the submatrix $M_u^{\bar{r}}$ cannot have rank greater than $N_c - 2$. It is easy to see that if we had taken M of rank $N_c - 1$ but with $M_u^{\bar{r}}$ of lesser rank, we would not have had the second of equation (2.8), and we would have found consistent solutions to the F-flatness conditions. When we study the $SU(2)$ case below, we will exhibit this explicitly.

3. Accidental Symmetries for Duals of $SU(2)$ Gauge Theories

When the gauge group is $SU(2)$, both mesons and baryons are quadratic in the underlying fields. Since the fundamental of $SU(2)$ is pseudoreal, there is no invariant distinction between Q and $\tilde{Q}$; as a result the flavor symmetry is enhanced to $SU(2N_f)$ and baryon perturbations (2.3) are simply mass terms. We will retain, for our present purposes, the usual $SU(N_c)$ labelling convention, which makes an artificial distinction between Q and $\tilde{Q}$. The gauge invariant bilinears, which transform as an antisymmetric tensor $V(Q)$ of $SU(2N_f)$, then break up into "mesons" and "baryons" as follows:

$$V(Q) = \begin{pmatrix} B(Q) & M(Q) \\ -M^T(Q) & \tilde{B}(\tilde{Q}) \end{pmatrix}. \tag{3.1}$$

As described in Ref. [1], for $N_f > 3$ the theory has a magnetic description using a gauge group $SU(N_f - 2)$, N_f flavors and a superpotential $W = \mu^{-1} q \cdot M \cdot \tilde{q}$, where M_u^r are gauge singlets which map to $Q^r \tilde{Q}_u$. Clearly this theory, at least in the UV, does not possess the full $SU(2N_f)$ symmetry.[5] However, in the IR theory (which is in a non-Abelian Coulomb or free magnetic phase, depending on N_f), the operators of interest are

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[5]   This theory has an alternate dual description which does explicitly preserve $SU(2N_f)$ invariance. The singlet fields and the assignment of charges to the q fields are different in this case, and the duality is a member of the Sp series of Refs. [1,8].

the gauge invariants M, $B(q)$, $\tilde{B}(\tilde{q})$ and $N = q\tilde{q}$. The latter is a redundant operator, and the remaining invariants transform in a representation of $SU(2N_f)$, as in Eq. (3.1).

We consider adding to the $SU(2)$ theory a superpotential of the form:

$$W_{\text{tree}} = m_r^u Q^r \tilde{Q}_u + \frac{1}{2} h_{rs} B^{rs}(Q) + \frac{1}{2} \tilde{h}^{uv} \tilde{B}_{uv}(\tilde{Q}). \tag{3.2}$$

The F-term conditions arising from (3.2) are

$$\begin{aligned}
\langle \frac{\partial W}{\partial Q^{\alpha r}} \rangle &= \langle m_r^u \tilde{Q}_{\alpha u} + \epsilon_{\alpha\beta} h_{rs} Q^{\beta s} \rangle = 0 \\
\langle \frac{\partial W}{\partial \tilde{Q}_{\alpha u}} \rangle &= \langle m_r^u Q^{\alpha r} + \epsilon^{\alpha\beta} \tilde{h}^{uv} \tilde{Q}_{\beta v} \rangle = 0
\end{aligned} \tag{3.3}$$

If we perform a rotation in $SU(2N_f)$ on this theory, the various terms in (3.2) will simply rotate into one another, and the physics is unchanged. However, such a transformation has a non-trivial effect on the magnetic theory away from the IR fixed point. We now explore this effect, looking at the simplest cases of m of rank one or h of rank one, and the interpolation between them.

3.1. $m \neq 0$

We consider first the case [1] where m has rank one and $h = \tilde{h} = 0$:

$$W_{\text{tree}} = m Q^1 \tilde{Q}_1 . \tag{3.4}$$

For large values of m, the fields Q^1, $\tilde{Q}_1$ are massive and can be integrated out. For $N_f > 4$, this procedure sets $\langle Q^1 \rangle = \langle \tilde{Q}_1 \rangle = 0$ and leads to a low-energy $SU(2)$ gauge theory with $N_f - 1$ flavors and no superpotential.

In the magnetic $SU(N_f - 2)$ gauge theory, the inclusion of (3.4) results in a superpotential

$$\tilde{W}_{\text{tree}} = \frac{1}{\mu} q_r M_u^{\ r} \tilde{q}^u + m M_1^1 . \tag{3.5}$$

Here the quarks q_1 and $\tilde{q}^1$ have a vacuum expectation value given by

$$\langle q_1 \cdot \tilde{q}^1 \rangle = -m\mu \tag{3.6}$$

from the F-flatness condition for M. This breaks the gauge symmetry down to $SU(N_f - 3)$ with $N_f - 1$ flavors, which is indeed dual [1] to the low-energy $SU(2)$ electric theory with $N_f - 1$ flavors. For $N_f - 1 > 5$ the duality is manifested through the flow of the magnetic theory to the free electric theory in the IR; for $N_f - 1 = 4$ or 5 both theories flow to the same interacting IR fixed point.

3.2. $h \neq 0$

We now consider the case where only $h_{12} \equiv h \neq 0$. From the point of view of the electric theory, this is a trivial flavor rotation $\tilde{Q}_1 \to Q^2$ of the case studied above. The equations (3.3) reduce to $\langle Q^{\alpha 1} \rangle = \langle Q^{\alpha 2} \rangle = 0$, and thus $\langle M_u^1(Q) \rangle = \langle M_u^2(Q) \rangle = \langle B^{1r}(Q) \rangle = \langle B^{2r}(Q) \rangle = 0$, where $r = 3, \ldots, N_f$ and $u = 1, \ldots, N_f$. Taking into account the D-flatness conditions, we find that $M_u^r(Q)$ can be at most rank 2, and $B(Q)$, $\tilde{B}(\tilde{Q})$ at most rank 1, with the constraint

$$M_u^r(Q)M_v^s(Q) - M_v^r(Q)M_u^s(Q) = B^{rs}(Q)\tilde{B}_{uv}(\tilde{Q}) . \tag{3.7}$$

We wish to show now that the same flat directions are found in the magnetic theory.

The magnetic theory is $SU(N_f - 2)$ with N_f flavors and a superpotential

$$W_{\text{tree}} = \frac{1}{\mu} q_r M_u^{\ r} \tilde{q}^u + h B^{12}(q). \tag{3.8}$$

Although h acts as a mass term in the electric theory, the physics in the magnetic dual is not that of symmetry breaking, in contrast to the case of non-zero m. Instead, the analysis of section 2 applies here and one easily finds the condition $\langle B^{1r} \rangle = \langle B^{2r} \rangle = 0$ as in (2.6). Furthermore, as in (2.7)–(2.8), the flat directions labelled by M_u^1 and M_u^2 are dynamically blocked. We now show that the remaining part of M must be rank 2 and satisfies the correct constraint (3.7).

Consider the direction where M_3^3 is non-zero. This vacuum expectation value gives mass to the fields $q_3, \tilde{q}^3$, which should be integrated out at low energy scales, leading to a low-energy theory with one more flavor than color. This theory confines and generates a dynamical superpotential for the confined fields $N_{\hat{r}}^{\hat{u}} = q_{\hat{r}} \tilde{q}^{\hat{u}}$ and $B_L^{\ \hat{r}} \propto B^{3\hat{r}}(q)$, $\tilde{B}_{L\hat{u}} \propto \tilde{B}_{3\hat{u}}(\tilde{q})$ (here, $\hat{r} = 1, 2, 4 \ldots N_f$)

$$\tilde{W}_L = \mu^{-1} M_{\hat{u}}^{\hat{r}} N_{\hat{r}}^{\hat{u}} - \frac{\mu}{M_3^3} \frac{\det N_{\hat{r}}^{\hat{u}}}{\tilde{\Lambda}^{\tilde{b}}} - \frac{B_L^{\ \hat{r}} N_{\hat{r}}^{\hat{u}} \tilde{B}_{L\hat{u}}}{\mu} - \frac{N_{\hat{r}}^{\hat{u}} M_3^{\hat{r}} M_{\hat{u}}^3}{\mu M_3^3} . \tag{3.9}$$

Compare this with (2.7), noting the absence of the hB term. The flatness conditions have solutions with $N = 0$ and $M_{\hat{u}}^{\hat{r}} = (B^{3\hat{r}} \tilde{B}_{3\hat{u}} + M_3^{\hat{r}} M_{\hat{u}}^3)/M_3^3$, which is one of the constraints (3.7) of the electric theory. A similar analysis holds if M is rank 2, for which the number of flavors and colors are equal. If we take M_3^3, M_4^4 nonzero, then instead of a dynamical superpotential there is a constraint [7]

$$\det(N_{\hat{r}}^{\hat{u}}) + \frac{(-\mu)^{N_f - 2}}{\Lambda^b} B\tilde{B} = \tilde{\Lambda}_L^{\tilde{b}_L} = M_3^3 M_4^4 \frac{\tilde{\Lambda}^{\tilde{b}}}{\mu^2} \tag{3.10}$$

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where $\hat{r} = 1, 2, 5 \ldots N_f$, $B = B^{34}$ and $\tilde{B} = \tilde{B}_{34}$. Here the only solution to the flatness conditions is $M_{\hat{u}}^{\hat{r}} = N_{\hat{r}}^{\hat{u}} = 0$, so Eq. (3.10) satisfies the constraint (3.7). Finally, for M of higher rank, the theory has fewer flavors than colors, and all flat directions are lifted by a dynamical superpotential [9,7]. One can also show that the baryons are at most rank one. Thus, the flat directions in the two theories are indeed the same.

3.3. $h, m \neq 0$

We may interpolate between the two cases studied above by letting both $m_1^1 = m$ and $h^{12} = h$ be non-zero. In the electric $SU(2)$ theory, a single flavor becomes massive. The requirements of Eq. (3.3) imply $M_j^1(Q) = B^{1j}(Q) = 0$ and

$$\langle m M_1^r(Q) + h B^{r2}(Q) \rangle = 0 \ , \tag{3.11}$$
$$\langle m \tilde{B}_{1u}(Q) + h M_u^2(Q) \rangle = 0 \ ;$$

thus these operators mix through perturbative dynamics.

In the magnetic theory, which is broken to $SU(N_f - 3)$, Eqs. (3.11) are also satisfied. The first equation can be seen immediately by considering the F-flatness conditions. The second equation arises in a more interesting way through non-perturbative effects. One way to see this is to take $M_2^2 \neq 0$; then q_2 and $\tilde{q}^2$ are massive and should be integrated out, resulting in a dynamically generated superpotential

$$\tilde{W}_L = m \hat{M}_1^1 + \mu^{-1} \hat{M} \hat{N} - \frac{1}{\mu M_2^2} \hat{N}_{\hat{r}}^{\hat{u}} M_2^{\hat{r}} M_{\hat{u}}^2 + h \hat{B}^1 - \frac{\det \hat{N}}{\tilde{\Lambda}^{\tilde{b}} M_2^2} - \frac{\hat{B}^r \hat{N}_r^u \tilde{\hat{B}}_u}{\mu M_2^2} \tag{3.12}$$

where hatted operators have indices running over $1, 3, 4, \ldots, N_f$ and where $\hat{B}^r = B^{r2}$. The F-flatness conditions imply $N_1^1 = -m\mu$; from $\partial W / \partial \hat{B}^1$ we find that

$$h M_2^2 + m \tilde{B}_{12} = 0 \tag{3.13}$$

Thus, in the magnetic theory, the mixing of the relevant operator $M_u^{\ r}$ and the IR-relevant operator $\tilde{B}(q)$ arises through a combination of perturbative and non-perturbative dynamics.

To summarize, as we rotate from a theory with $m \neq 0$ to one with $h \neq 0$ (a trivial symmetry operation in the electric $SU(2)$ theory), we find that the magnetic theory changes in an intricate way. From $h = 0$, where it is an $SU(N_f - 3)$ gauge theory with $N_f - 1$ flavors and no special dynamics, the theory changes to one in which singlet meson and composite

baryon operators mix dynamically, until finally for $m = 0$ the $SU(N_f - 2)$ symmetry is restored along with all N_f flavors, and with gauge dynamics and the superpotential $W_{\text{tree}} = hB(q)$ restricting the flat directions. Remarkably, this continuous transformation of the UV theory does not in any way change the low-energy dynamics, on which it is realized as a symmetry transformation of the low-energy fields.

The generalization to mass matrices and h matrices of higher rank is straightforward.

3.4. Symmetry Breaking in the $SU(2)$ Theory

It is also interesting to consider the effect of giving a baryon operator a vacuum expectation value. Let us first review the effect of $\langle Q^1 \tilde{Q}_1 \rangle \neq 0$ which was studied in detail in Ref. [1]. The gauge symmetry is broken completely; the D-flatness conditions require that M_u^r have rank at most 2, in which case (after using the remaining flavor symmetry) we can write a constraint

$$M_1^1(Q)M_2^2(Q) - M_2^1(Q)M_1^2(Q) = B^{12}(Q)\tilde{B}_{12}(\tilde{Q}) . \tag{3.14}$$

With a general mass matrix $\hat{m}_{\hat{r}}^{\hat{u}} Q^{\hat{r}} \tilde{Q}_{\hat{u}}$ for the other fields instanton dynamics generates a superpotential

$$W_L = \Lambda^b \frac{\det \hat{m}}{Q^1 \tilde{Q}_1} \tag{3.15}$$

The magnetic theory with $\langle M_1^1 \rangle \neq 0$ is $SU(N_f - 2)$ with $N_f - 1$ flavors. Its associated dynamical superpotential

$$\mu^{-1}\hat{M}\hat{N} - \frac{\mu \det \hat{N}}{\tilde{\Lambda}^{\tilde{b}} M_1^1} - \frac{\hat{B}\hat{N}\hat{\tilde{B}}}{\mu M_1^1} \tag{3.16}$$

leads to (3.14) through $\partial W/\partial N_2^2$ and to (3.15) when $\hat{m}\hat{M}$ is added to the superpotential.

Now we wish to rotate $\tilde{Q}_1 \leftrightarrow Q^2$ so that $B^{12}(Q)$ has a vacuum expectation value. The flavor-rotated version of (3.14) is

$$M_u^1(Q)B^{2r}(Q) + M_u^2(Q)B^{r1}(Q) + M_u^r(Q)B^{12}(Q) = 0 . \tag{3.17}$$

In the magnetic $SU(N_f - 2)$ theory we have

$$\langle B^{12}(q) \rangle \propto \langle q_3 q_4 \cdots q_{N_f - 2} \rangle \neq 0 \tag{3.18}$$

which completely breaks the $SU(N_f - 2)$ gauge group. If we now let $M_3^s \neq 0$ for $s = 1, 2, 3$ we find the F-flatness conditions

$$\langle \tilde{q}^3 \rangle = 0 ; \quad \sum_{s=1}^{3} \langle M_3^s q_s \rangle = 0 \tag{3.19}$$

Multiplying the latter equation by $\langle q_4 \cdots q_{N_c} \rangle$ we find (3.17) for $r = u = 3$, *etc.*

If we add to the electric superpotential $\tilde{h}\tilde{Q}_1\tilde{Q}_2 + \bar{m}^{\bar{u}}_{\bar{r}}Q^{\bar{r}}\tilde{Q}_{\bar{u}}$, $\bar{r}, \bar{u} = 3, 4, \ldots, N_f$, we get a rotated version of (3.15).

$$W_L = -\Lambda^b \frac{\tilde{h}\det\bar{m}}{Q^1 Q^2} \tag{3.20}$$

In the magnetic theory the superpotential is

$$\mu^{-1}M^{\bar{r}}_{\bar{u}}N^{\bar{u}}_{\bar{r}}(q) + \tilde{h}\tilde{B}_{12}(\tilde{q}) + \bar{m}^{\bar{u}}_{\bar{r}}M^{\bar{r}}_{\bar{u}} \ . \tag{3.21}$$

The F-term equations give $q_{\bar{r}}\tilde{q}^{\bar{u}} = -\mu\bar{m}^{\bar{u}}_{\bar{r}}$. Meanwhile, the D-term equations imply that $\det_{\bar{r},\bar{u}} q_{\bar{r}}\tilde{q}^{\bar{u}} = -\Lambda^{-b}(-\mu)^{N_f - 2}B^{12}(q)\tilde{B}_{12}(\tilde{q})$. Substituting these relations into the superpotential (3.21), one finds the first and third terms cancel while the second term becomes

$$\tilde{W}_L = \tilde{h}\tilde{B}_{12}(\tilde{q}) = -\Lambda^b \frac{\tilde{h}\det\bar{m}}{B^{12}} \tag{3.22}$$

which is the same as (3.20). Note that the argument is now perturbative in the magnetic theory, in contrast to the case of meson expectation values.

As in section 3.3, one can again continuously connect these theories; we will not present the details. The key result is that the IR dynamics of the magnetic $SU(N_f - 2)$ gauge theory with $N_f - 1$ flavors is exactly equivalent to that of a completely broken $SU(N_f - 2)$ gauge theory with a certain superpotential, and that an infinite class of theories interpolates between them.

3.5. $SU(2)$ with $N_f = 4$ and its dual

This theory is special, as it is dual to a theory with the same gauge group and number of doublet representations; however the magnetic theory contains extra singlets and a superpotential $W = \mu^{-1}q \cdot M \cdot \tilde{q}$ which breaks its flavor symmetry from $SU(8)$ to $SU(4) \times SU(4) \times U(1)$. If we give mass to any pair of doublets, so that six remain, we immediately generate a dynamical superpotential [7]. We will see interesting physics from this below.

We begin by reviewing the theory with an ordinary mass term $mQ^1\tilde{Q}_1$. At scales below this mass, we integrate out the massive doublets and obtain a theory which confines and generates a dynamical superpotential:

$$W_L = \frac{1}{m\Lambda^2}\left(\hat{B} \cdot \hat{M} \cdot \hat{\tilde{B}} - \det\hat{M}\right). \tag{3.23}$$

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In the magnetic theory, m leads to symmetry breaking; in this case, the superpotential is generated [1] partly at tree level, and partly by instantons in the broken group. The tree level part is obtained by expanding around the vacuum expectation values:

$$\tilde{W} = \mu^{-1} M_u^r q_r \tilde{q}^u + m M_1^1 \quad \rightarrow \quad \tilde{W}_{L,tree} = \mu^{-1} M_{\hat{u}}^{\hat{r}} (q_2)_{\hat{r}} (\tilde{q}^2)^{\hat{u}} \tag{3.24}$$

where $(q_2)_{\hat{r}}$ is the second color component of $q_{\hat{r}}$, and we have chosen the vevs along the first color direction so that the first components of $q_r, \tilde{q}^u$ are massive. By identifying

$$(q_2)_{\hat{r}} = \frac{1}{v} \sqrt{\frac{-\mu^2}{\Lambda^2}} B^{\hat{s}\hat{t}} \frac{\epsilon_{1\hat{r}\hat{s}\hat{t}}}{2!} \quad ; \quad (\tilde{q}^2)^{\hat{u}} = \frac{1}{v} \sqrt{\frac{-\mu^2}{\Lambda^2}} \tilde{B}_{\hat{v}\hat{w}} \frac{\epsilon^{1\hat{u}\hat{v}\hat{w}}}{2!} \tag{3.25}$$

where $v^2 = -m\mu$, we may rewrite this superpotential as

$$\tilde{W}_{L,tree} = \frac{1}{m\Lambda^2} \frac{1}{2!} \epsilon_{\hat{r}\hat{s}\hat{t}} B^{\hat{s}\hat{t}} M_{\hat{u}}^{\hat{r}} \frac{1}{2!} \epsilon^{\hat{u}\hat{v}\hat{w}} \tilde{B}_{\hat{v}\hat{w}}. \tag{3.26}$$

The remaining term comes from instantons in the unbroken group; we can uncover this term by turning on vevs for $\hat{M}$.[1] This will give mass to three of the four flavors; the low-energy theory in this case contains an instanton superpotential: [9]

$$\tilde{W}_{L,dyn} = \frac{\tilde{\Lambda}_{L'}^5}{q_1 \tilde{q}^1} = -\tilde{\Lambda}^2 \frac{\det \hat{M}}{m\mu^4} = -\frac{\det \hat{M}}{\Lambda^2 m} \ . \tag{3.27}$$

Thus the terms (3.26) and (3.27) combine to reproduce Eq. (3.23) for the low-energy magnetic theory.

Consider now the theory with $W_{\text{tree}} = hQ^1Q^2$. The F-term equations tell us that $M_u^1 = M_u^2 = B^{1r} = B^{2r} = 0$. Integrating out the massive quarks gives us a superpotential

$$W_L = \frac{1}{h\Lambda^2} \left(M_u^r M_v^s - \frac{1}{4} B^{rs} \tilde{B}_{uv} \right) \tilde{B}_{wx} \frac{\epsilon^{uvwx}}{2!} \frac{\epsilon_{rs}}{2!} \tag{3.28}$$

which is just an $SU(8)$ rotation of Eq. (3.23).

In the magnetic theory, we have a superpotential

$$\tilde{W}_{\text{tree}} = \sqrt{\frac{-\Lambda^2}{\mu^2}} h q_3 q_4 + \mu^{-1} \sum_{a=1,2} q_a M_u^a \tilde{q}^u + \mu^{-1} \sum_{i=3,4} q_i M_u^i \tilde{q}^u. \tag{3.29}$$

We see that $h \neq 0$, in contrast to the case for $m \neq 0$, does not lead to symmetry breaking; rather, for large h, q_3, q_4 are massive. Integrating them out, we find a superpotential

$$\tilde{W}_{L,\text{tree}} = -\sqrt{\frac{-\mu^2}{\Lambda^2}} \frac{1}{h\mu^2} \tilde{q}^u M_u^3 M_v^4 \tilde{q}^v + \mu^{-1} q_a M_u^a \tilde{q}^u \tag{3.30}$$

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where $a = 1, 2$ and $u, v = 1, \ldots, 4$. Now because this theory has only three flavors, it confines, leading to a superpotential:

$$\tilde{W}_L = +\frac{1}{h\Lambda^2}\frac{\epsilon_{rs}}{2!}\frac{\epsilon^{uvwx}}{2!}M_u^r M_v^s \tilde{B}_{wx} + \mu^{-1}N_a^u M_u^a$$
$$- \frac{\mu^2}{h\Lambda^2\tilde{\Lambda}^2}\left(\tilde{B}_{wx}N_a^w N_b^x + \frac{\mu^2}{4\Lambda^2}B_{ab}\tilde{B}_{uv}\tilde{B}_{wx}\frac{\epsilon^{uvwx}}{2!}\right)\frac{\epsilon^{ab}}{2!} \tag{3.31}$$

The first two terms are those of Eq. (3.30) and the last is generated dynamically; using $\Lambda^2\tilde{\Lambda}^2 = \mu^4$ one sees that it is again merely an $SU(8)$-flavor rotation of the analogous term (3.23), similar to that employed in writing Eq. (3.28). From the F-term equations, we see that we must have $\langle M_u^a \rangle = \langle B^{ar} \rangle = 0$, in agreement with the electric theory; also we have $\langle N \rangle = 0$, so that (3.28) and (3.31) agree.

3.6. $SU(2)$ with $N_f = 5$ and its dual

As discussed in [1], the superpotential $mQ^1\tilde{Q}_1$ causes this theory to flow to $SU(2)$ with four flavors, which flows to an interacting fixed point in the IR. The flavor symmetries ensure that a term hQ^1Q^2 will have the same effect. In the magnetic $SU(3)$ gauge theory this means that the superpotential $W \propto q_3 q_4 q_5$, which makes the theory chiral, still leads to a theory whose dynamics we know.

Let us examine the magnetic theory in some detail. It has gauge group $SU(3)$ and a superpotential

$$W = h\sqrt{\frac{\Lambda}{\mu^3}}q_3 q_4 q_5 + \mu^{-1}M_u^r q_r \tilde{q}^u$$

which leaves a global symmetry $SU(2) \times SU(3) \times SU(5) \times U(1)_R$. In the IR, we have operators $B^{\hat{r}\hat{s}}$, $B^{\bar{r}\bar{s}}$, $B^{\hat{r}\bar{s}}$, $\tilde{B}_{uv}$, $M_u^{\bar{r}}$, $M_u^{\hat{r}}$, $N_{\hat{r}}^u$ and $N_{\bar{r}}^u$, where $\hat{r} = 1, 2$, $\bar{r} = 3, 4, 5$, and $u = 1, \ldots, 5$. From the F-term equations, we find $N_{\hat{r}}^u = N_{\bar{r}}^u = B^{\hat{r}\hat{s}} = B^{\hat{r}\bar{s}} = 0$ and thus these operators are removed in the IR. In addition, the flat directions associated with $M_u^{\hat{r}}$ are dynamically blocked. If $\langle M_u^{\hat{r}} \rangle$ has rank 1, then one flavor is given a mass; this leaves $SU(3)$ with 4 flavors, which confines and, as in Eqs. (2.7)-(2.8), generates a superpotential with no supersymmetric ground state. The remaining 28 fields $B^{\bar{r}\bar{s}}$, $M_u^{\bar{r}}$ and $\tilde{B}_{uv}$ have R-charge 1 and satisfy the F-term equations

$$M_u^{\bar{r}}B^{\bar{s}\bar{t}}\epsilon_{\bar{r}\bar{s}\bar{t}} = 0 \; ; \quad M_u^{\bar{r}}\tilde{B}_{vw}\epsilon^{uvwxy} = 0. \tag{3.32}$$

This is in agreement with the low-energy electric theory, which is $SU(2)$ with 4 flavors; its invariants consist of 28 fields V_{ij} with R-charge 1 in the antisymmetric representation

of the $SU(8)$ flavor symmetry. In the electric theory the equations (3.32) arise as classical constraints, which are unmodified in the quantum theory.

We emphasize that the $SU(3)$ magnetic gauge theory is chiral, but its infrared physics is fully under control. Indeed, it is perhaps the simplest (somewhat trivial) example of a chiral theory which flows to a fixed point with a dual description in terms of a non-chiral theory.

4. Various Remarks

In this section we note a number of related implications of our work.

4.1. Other Examples of Accidental Symmetries

Many other examples of non-perturbative accidental symmetries are known, and are too numerous to list in their entirety. For example, the theories of $SU(N_c)$ which appear in Refs. [5,10] all have these properties for $N_c = 2$. Other examples include $Spin$ group dualities of [11]. If one breaks $Spin(7)$ with $N_f + 2$ spinors to $SU(3)$ with N_f flavors, the low-energy theory has an $SU(N_f) \times SU(N_f) \times U(1)$ global symmetry. However, the dual $SU(N_f - 2)$ gauge theory, with a symmetric tensor and $N_f + 2$ fields in the antifundamental representation, has only $SU(N_f) \times U(1)$ symmetry in the UV.

Another interesting example has recently emerged in the context of three-dimensional N=4 gauge theories [12]. Mirror symmetry relates two theories, with Higgs and Coulomb branches, hypermultiplets and vector multiplets, masses and Fayet-Iliopoulos couplings exchanged. In general, the mass terms appear in a larger flavor symmetry than the Fayet-Iliopoulos couplings, and the latter transform properly under the larger symmetry only in the IR. The non-perturbative dynamics involved in this result should be quite interesting.

4.2. Ultraviolet Irrelevant and Infrared Relevant Operators

In the remainder of this section we focus our attention on the special behavior of baryon operators in the vacua of $SU(N_c)$ theories at the origin of moduli space, where classically the full gauge group is unbroken, and where quantum mechanically one often expects conformal field theories in the IR.

One of the common features of N=1 duality as is the presence of operators in the IR whose dimensions are far different from their canonical values. The baryons in the above theories are of this type. In general, the baryon operator which we add to the magnetic

superpotential (3.8) is of dimension $N_f - 2$ in the UV, which is perturbatively marginal for $N_f = 5$ and is irrelevant (thereby making the theory perturbatively non-renormalizable) for $N_f > 5$.[6] However, in the IR this baryon operator is a relevant operator. For $N_f = 3, 4, 5$ the theory flows to an interacting fixed point; the superconformal algebra tells us the baryon has dimension $1, \frac{3}{2}, \frac{9}{5}$ [1]. For $N_f > 5$ the theory flows to a free $SU(2)$ gauge theory, whose baryons have dimension 2.

To say this another way, one might naively have the impression from perturbation theory that adding a high-dimension baryon to the superpotential would not affect the IR behavior of the theory. As we have seen, however, such operators can indeed play a crucial role, even changing the phase of the theory. For example, consider the magnetic $SU(5)$ theory with 7 flavors and mesons that flows in the IR to a weakly-coupled electric $SU(2)$ gauge theory with 7 flavors. Adding to this theory's superpotential a dimension-five baryon operator with coefficient $h^{r_1 r_2}$ of rank 1 removes one flavor in the low-energy effective $SU(2)$ theory without changing the phase. However, if h has rank 2 then the $SU(5)$ theory hits a fixed point at finite coupling (which is physically equivalent to the theory of $SU(2)$ with $N_f = 5$), while if it has rank 4 the theory flows to strong coupling, confines, and has massless mesons and baryons. Other dramatic effects of similar operators (known as "dangerously irrelevant operators"; see the appendix of [5] for a brief discussion) have been observed elsewhere [3,5,10,13,14].

While we so far have only explored the physics of $SU(2)$ theories with $2N_f$ doublets and their magnetic duals, there are other interesting $SU(N_c)$ theories with IR relevant baryons, which we will explore further in the following sections. Before beginning, we list the various possibilities.

The dimensions of baryons are the same for theories with $SU(N_c)$ and $SU(N_f - N_c)$ gauge groups. These dimensions are determined by the R charges of their constituents. Since these charges depend in these theories only on the $U(1)_R$-gauge anomalies and not on the presence or absence of gauge singlet mesons, a theory with gauge group $SU(N_c)$ (with a dual representation as $SU(N_f - N_c)$ with singlets) has a baryon of the same IR dimensions as an $SU(N_f - N_c)$ theory (with a dual representation as $SU(N_c)$ with singlets.)

In particular, theories with $SU(N_f - 2)$ gauge group and *no* mesons M_u^r therefore have baryons at low-energy with the same dimension as those with $SU(2)$ gauge group,

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[6] Recall that the superpotential in four dimensions has dimension 3, so baryons of dimension greater than 3 have couplings with inverse mass dimensions and are irrelevant at weak coupling.

the same number of flavors, and *no* mesons $M_u{}^r$. Another easy case is $SU(N_f - 1)$, for which the dual has no gauge group; the low-energy theory always has the baryon as a fundamental field of dimension 1 [7]. In all of these theories, addition of baryon operators to the superpotential may be easily analyzed, since these operators are either linear or bilinear in the fields in at least one description of the theory.

However, there are other examples where the baryons are neither linear nor bilinear in any description. These include $SU(3)$ for $3 < N_f$, and by association $SU(N_f - 3)$ theories; for $N_f > 8$ the baryons have their canonical dimension (namely 3) at the free $SU(3)$ IR gauge theory, while for $N_f = 4, 5, 6, 7, 8$ the baryons have IR dimension $1, 2, \frac{9}{4}, \frac{18}{7}, \frac{45}{16}$. $SU(4)$ with $N_f = 8$ has a strongly coupled fixed point with a baryon which is IR-marginal (though not, in general, exactly marginal.)

In all other $SU(N_c)$ gauge theories with N_f flavors, the baryons are irrelevant in the IR, so adding them to the superpotential has no effect on the conformal theory at the origin of moduli space.

4.3. Baryons in $SU(N_f - 2)$ Gauge Theories and New Fixed Points

Previously our focus was on non-perturbative accidental symmetries, so we have considered as the electric theory only the theory of $SU(2)$ with $2N_f$ doublets. But we may also consider the physics of $SU(N_f - 2)$ with N_f flavors and no singlets as the electric theory. In this case, both this theory and its dual have the usual $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ flavor symmetry and there are no accidental IR symmetries. Nevertheless, the baryon operators are dual to mass terms in the $SU(2)$ magnetic theory and one can analyze these theories in some detail.

In particular, one is immediately led to a number of new fixed points, some of which are certainly present, others of which are likely to be present. We noted in a previous section that $SU(2)$ with 10 doublets, two of which are massive, is dual to an $SU(3)$ magnetic theory with $N_f = 5$ and a single baryon operator in its superpotential; both theories flow in the IR to a fixed point characteristic of $SU(2)$ with 8 doublets. We may now consider the behavior of $SU(3)$ with $N_f = 5$ as the electric theory. The effect of a superpotential $W = hQ^1Q^2Q^3$ is to change the dual superpotential to $\tilde{W} = \mu^{-1}(q_r M_u{}^r \tilde{q}^u + h\Lambda^2 q_4 q_5)$. Integrating out the massive fields q_4, q_5 leaves

$$\tilde{W} = \sum_{r=1}^{3} \sum_{u=1}^{5} \mu^{-1} q_r M_u{}^r \tilde{q}^u \tag{4.1}$$

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plus terms proportional to $1/h$ that are irrelevant in the IR. The IR behavior of this chiral $SU(2)$ gauge theory is not known but it is likely that it flows to a fixed point. The properties of this fixed point are not exactly determined; the symmetries are broken enough that the R charges of the q and $\tilde{q}$ fields, and therefore their dimensions, cannot be determined independently.

More generally, if we consider $SU(N_f - 2)$ with N_f flavors as the electric theory with $W = hB + \tilde{h}\tilde{B}$, with h of rank p and $\tilde{h}$ of rank $\tilde{p}$, the dual theory is $SU(2)$ with $2(N_f - p - \tilde{p})$ doublets and $(N_f - 2p)(N_f - 2\tilde{p})$ mesons $M_u^{\ r}$ coupled as

$$\tilde{W} = \sum_{r=1}^{N_f - 2p} \sum_{u=1}^{N_f - 2\tilde{p}} \mu^{-1} q_r M_u^{\ r} \tilde{q}^u \tag{4.2}$$

For $2(N_f - p - \tilde{p}) \geq 12$, these theories are free in the IR and their behavior is therefore straightforward. For $2(N_f - p - \tilde{p}) = 8$ or 10 the situation is more interesting. If $p = \tilde{p}$, $N_f - 2p = 0$, or $N_f - 2\tilde{p} = 0$, these theories are already known [1] to reach interacting fixed points, at which the dimensions of all chiral operators can be determined. For other cases the theories have not been considered. However it seems plausible that all of them reach new, previously unidentified interacting fixed points. Note that these theories are chiral and the global symmetries are insufficient to determine the dimensions of most chiral operators. For $2(N_f - p - \tilde{p}) < 8$ these theories probably confine and generate dynamical superpotentials, which we have not analyzed.

4.4. Other New Fixed Points Associated with Baryon Perturbations

For those theories with IR-relevant baryon operators which are not dual to mass terms, it is impossible to analyze their non-perturbative effects in detail. Nonetheless, as they certainly cause the theory to flow away from its original fixed point, it is worth speculating about the physics of this process. For theories with IR-marginal baryon operators, the question is whether the operators are exactly marginal, marginally relevant or marginally irrelevant. We will argue that in most cases they are expected to be marginally irrelevant, but, as shown in [13], in two independent cases they are exactly marginal.

Let us first consider theories with $SU(3)$ gauge group. For $N_f = 4, 5$ the discussion has already been covered. For $N_f = 6, 7, 8$ we may consider the effects of adding to the superpotential $hQ^1Q^2Q^3$; the resulting chiral theory probably flows to a previously unidentified fixed point, in analogy to the $N_f = 5$ case. We may also consider baryon terms of higher rank. While there is quite a bit of interesting physics in such an analysis,

the most interesting case is that of $N_f = 6$ with $W = hQ^1Q^2Q^3 + h'Q^4Q^5Q^6$. We may expect a basin of attraction in which $h = h'$ in the infrared; if this theory reaches a fixed point then it has enough symmetry that despite its chiral nature all the R charges may be determined. In particular, the R charges of the six Q^r are all $\frac{2}{3}$ (thus they have dimension 1) while those of the $\tilde{Q}_u$ are $\frac{1}{3}$ (dimension $\frac{1}{2}$.)

For $SU(3)$ with $N_f \geq 9$, the theory is not asymptotically free and one may easily check that the baryons are perturbatively irrelevant at weak coupling. However [15,13], there is a well-known exception for $N_f = 9$. The superpotential

$$W = h(Q^1Q^2Q^3 + Q^4Q^5Q^6 + Q^7Q^8Q^9 + \tilde{Q}_1\tilde{Q}_2\tilde{Q}_3 + \tilde{Q}_4\tilde{Q}_5\tilde{Q}_6 + \tilde{Q}_7\tilde{Q}_8\tilde{Q}_9) \qquad (4.3)$$

combined with a non-zero gauge coupling g actually represents an exactly marginal perturbation of the free fixed point; that is, there is a complex curve $F(g, h) = 0$ passing through $g = h = 0$ where this theory is conformal. One may see hints of this by looking in first-order perturbation theory; however this result holds to all orders [16] and in fact holds non-perturbatively [13].

We omit the discussion of $SU(N_f - 3)$, since the analysis is almost identical to that of $SU(3)$ theories with N_f flavors. We merely note that $SU(6)$ with $N_f = 9$ has an exactly marginal operator which is dual to that of Eq. (4.3) [13].

Finally we consider $SU(4)$ with $N_f = 8$. This theory has a non-trivial IR fixed point where its baryon becomes marginal. However it is certainly not exactly marginal (and is probably marginally irrelevant, in analogy to the case of $SU(3)$ with $N_f = 9$ considered above) unless one considers a superpotential of high symmetry, in analogy to (4.3) [13]

$$W = h(Q^1Q^2Q^3Q^4 + Q^5Q^6Q^7Q^8 + \tilde{Q}_1\tilde{Q}_2\tilde{Q}_3\tilde{Q}_4 + \tilde{Q}_5\tilde{Q}_6\tilde{Q}_7\tilde{Q}_8) \ . \qquad (4.4)$$

This operator (combined with a variation in the gauge coupling) is exactly marginal, even though it is irrelevant in the UV and makes the theory naively non-renormalizable.

5. Conclusion

We may summarize our main result formally as follows. It may happen that while a theory T_E has a global symmetry G_E in the UV which is preserved in the IR, its magnetic dual T_M has a smaller global symmetry G_M in the UV, and this global symmetry is enhanced to G_E only in the IR. In this case the theory T_M is actually only one of a

space of theories which share the same infrared behavior with T_E; this space has the form G_E/G_M. Trivial global rotations by elements of G_E in the electric theory are manifested as non-trivial translations inside the space of magnetic theories G_E/G_M.

In the specific example studied here, the effect of the $SU(2N_f)$ global symmetry of the electric $SU(2)$ theory can be easily observed by perturbing the theory. Each perturbation generates a continuously infinite class of magnetic duals, parametrized by a subspace of $SU(2N_f)/(SU(N_f) \times SU(N_f) \times U(1))$ depending on the perturbation. These classes contain magnetic theories with different gauge and flavor groups at the UV cutoff, all of which flow to the same IR dynamics. Unlike previously known examples of accidental symmetries, the physics which makes this possible involves non-perturbative dynamics. This is a general property of duality in the presence of accidental symmetries.

One noteworthy aspect of this physics is that it depends on the fact that certain baryons which are of high dimension in the UV become relevant in the IR through the effects of strong coupling. We have briefly discussed additional implications of this phenomenon, including new examples of phase transitions due to naively irrelevant operators, new chiral fixed points, and exactly marginal operators of high canonical dimension.

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